Spacetime foam, Casimir energy and black hole pair creation

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(February 5, 2008)

Abstract

We conjecture that the neutral black hole pair production is related to the vacuum fluctuation of pure gravity via the Casimir-like energy. Implications on the foam-like structure of spacetime are discussed.

I. INTRODUCTION

It was J. A. Wheeler who first conjectured that spacetime could be subjected to topology fluctuation at the Planck scale [1]. This means that spacetime undergoes a deep and rapid transformation in its structure. This changing spacetime is best known as "*spacetime foam*", which can be taken as a model for the quantum gravitational vacuum. Some authors have investigated the effects of such a *foamy* space on the cosmological constant, one example is the celebrated Coleman mechanism involving wormholes [2]. Nevertheless, how to realize such a foam-like space is still unknown as too is whether this represents the real quantum gravitational vacuum. For this purpose, we begin to consider the "simplest" quantum process

that could approximate the foam structure in absence of matter fields, that is the black hole pair creation. Different examples are known on this subject. The first example involves the study of black hole pair creation in a background magnetic field represented by the Ernst solution [3] which asymptotically approaches the Melvin metric [4]. Another example is the Schwarzschild-deSitter metric (SdS) which asymptotically approaches the deSitter metric. The extreme version is best known as the Nariai metric [5]. In this case the background is represented by the cosmological constant Λ acting on the neutral black hole pair produced, accelerating the components away from each other. Finally another example is given by the Schwarzschild metric which asymptotically approaches the flat metric and depends only on the mass parameter M. Metrics of this type are termed asymptotically flat (A.F.). Another metric which has the property of being A.F. is the Reissner-Nordström metric, which depends on two parameters: the mass M and the charge Q of the electromagnetic field. Nevertheless, all the cases mentioned above introduce an external background field like the magnetic field or the cosmological constant to produce the pair and accelerate the components far away. In this letter, we would like to consider the same process without the contribution of external fields, except gravity itself and consider the possible implications on the foam-like structure of spacetime. This choice linked to the vacuum Einstein's equations leads to the Schwarzschild and the flat metrics, where only the simplest case is considered, i.e. metrics which are spherically symmetric. Since the A.F. spacetimes are non-compact a subtraction scheme is needed to recover the correct equations under the constraint of fixed induced metrics on the boundary [6,7].

II. QUASILOCAL ENERGY AND ENTROPY IN PRESENCE OF A BIFURCATION SURFACE

Although it is not necessary for the forthcoming discussions, let us consider the maximal analytic extension of the Schwarzschild metric, i.e., the Kruskal manifold whose spatial slices Σ represent Einstein-Rosen bridges with wormhole topology $S^2 \times R^1$. Following Ref. [6],

the complete manifold $\mathcal{M}$ can be taken as a model for an eternal black hole composed of two wedges $\mathcal{M}_+$ and $\mathcal{M}_-$ located in the right and left sectors of a Kruskal diagram. The hypersurface Σ is divided in two parts Σ_+ and Σ_- by a bifurcation two-surface S_0. On Σ we can write the gravitational Hamiltonian

$$H_p = H - H_0 = \frac{1}{2\kappa}\int_\Sigma d^3x(N\mathcal{H}+N^i\mathcal{H}_i)$$

$$+\frac{1}{\kappa}\int_{S_+} d^2xN\sqrt{\sigma}\left(k-k^0\right) - \frac{1}{\kappa}\int_{S_-} d^2xN\sqrt{\sigma}\left(k-k^0\right), \tag{1}$$

where $\kappa = 8\pi G$. The Hamiltonian has both volume and boundary contributions. The volume part involves the Hamiltonian and momentum constraints

$$\mathcal{H} = (2\kappa)\, G_{ijkl}\pi^{ij}\pi^{kl} - \sqrt{^3g}R/\left(2\kappa\right) = 0,$$

$$\mathcal{H}_i = -2\pi^j_{i|j} = 0, \tag{2}$$

where $G_{ijkl} = \left(g_{ik}g_{jl} + g_{il}g_{jk} - g_{ij}g_{kl}\right)/\left(2\sqrt{g}\right)$ and R denotes the scalar curvature of the surface Σ. The volume part of the Hamiltonian (1) is zero when the Hamiltonian and momentum constraints are imposed. However, for the flat and the Schwarzschild space, constraints are immediately satisfied, then in this context the total Hamiltonian reduces to

$$H_p = \frac{1}{\kappa}\int_{S_+} d^2xN\sqrt{\sigma}\left(k-k^0\right) - \frac{1}{\kappa}\int_{S_-} d^2xN\sqrt{\sigma}\left(k-k^0\right). \tag{3}$$

Quasilocal energy is defined as the value of the Hamiltonian that generates unit time translations orthogonal to the two-dimensional boundaries, i.e.

$$E_{tot} = E_+ - E_-,$$

$$E_+ = \frac{1}{\kappa}\int_{S_+} d^2x\sqrt{\sigma}\left(k-k^0\right)$$

$$E_- = -\frac{1}{\kappa}\int_{S_-} d^2x\sqrt{\sigma}\left(k-k^0\right). \tag{4}$$

where $|N| = 1$ at both S_+ and S_-. E_{tot} is the quasilocal energy of a spacelike hypersurface $\Sigma = \Sigma_+ \cup \Sigma_-$ bounded by two boundaries ${}^3S_+$ and ${}^3S_-$ located in the two disconnected regions M_+ and M_- respectively. We have included the subtraction terms k^0 for the energy. k^0 represents the trace of the extrinsic curvature corresponding to embedding in the two-dimensional boundaries ${}^2S_+$ and ${}^2S_-$ in three-dimensional Euclidean space. Let us consider the case of the static Einstein-Rosen bridge whose metric is defined as:

$$ds^2 = -N^2 dt^2 + g_{yy} dy^2 + r^2(y)\, d\Omega^2, \tag{5}$$

where N, g_{yy}, and r are functions of the radial coordinate y continuously defined on $\mathcal{M}$, with $dy = dr/\sqrt{1 - \frac{2m}{r}}$. If we make the identification $N^2 = 1 - \frac{2m}{r}$, the line element (5) reduces to the S metric written in another form. The boundaries ${}^2S_+$ and ${}^2S_-$ are located at coordinate values $y = y_+$ and $y = y_-$ respectively. The normal to the boundaries is $n^\mu = (h^{yy})^{\frac{1}{2}}\, \delta^\mu_y$. Since this normal is defined continuously along Σ, the value of k depends on the function $r_{,y}$, which is positive for ${}^2B_+$ and negative for ${}^2B_-$. The application of the quasilocal energy definition gives

$$E = E_+ - E_-$$

$$= \left(r\, |r_{,y}| \left[1 - (h^{yy})^{\frac{1}{2}} \right] \right)_{y=y_+} - \left(r\, |r_{,y}| \left[1 - (h^{yy})^{\frac{1}{2}} \right] \right)_{y=y_-}. \tag{6}$$

It is easy to see that E_+ and E_- tend individually to the $\mathcal{ADM}$ mass $\mathcal{M}$ when the boundaries ${}^3B_+$ and ${}^3B_-$ tend respectively to right and left spatial infinity. It should be noted that the total energy is zero for boundary conditions symmetric with respect to the bifurcation surface, i.e.,

$$E = E_+ - E_- = M + (-M) = 0, \tag{7}$$

where the asymptotic contribution has been considered. The same behaviour appears in the entropy calculation for the physical system under examination. Indeed

$$S_{tot} = S_+ - S_- = \exp\left(\frac{A^+}{4} - \frac{A^-}{4} \right) \simeq \exp\left(\frac{A_H}{4} - \frac{A_H}{4} \right) = \exp(0)\,, \tag{8}$$

where A^+ and A^- have the same meaning as E_+ and E_-. Note that for both entropy and energy this result is obtained at zero loop. We can also see eqs. (7) and (8) from a different point of view. In fact these eqs. say that flat space can be thought of as a composition of two pieces: the former, with positive energy, in the region Σ_+ and the latter, with negative energy, in the region Σ_-, where the positive and negative concern the bifurcation surface (hole) which is formed due to a topology change of the manifold. The most appropriate mechanism to explain this splitting seems to be a black hole pair creation.

III. BLACK HOLE PAIR CREATION

The formation of neutral black hole pairs with the two holes residing in the same universe is believed to be a highly suppressed process, at least for $\Lambda \gg 1$ in Planck's units [8]. The metric which describes such pair creation is the Nariai metric. When the cosmological constant is absent the SdS metric is reduced to the Schwarzschild metric which concerns a single black hole. However, one could regard each single Schwarzschild black hole in our universe as a mere part of a neutral pair, with the partner residing in the other universe. In this case the whole spacetime can be regarded as a black-hole pair formed up by a black hole with positive mass M in the coordinate system of the observer and an *anti black-hole* with negative mass $-M$ in the system where the observer is not present. From the instantonic point of view, one can represent neutral black hole pairs as instantons with zero total energy. An asymptotic observer in one universe would interpret each such pair as being formed by one black hole with positive mass M. What such an observer would actually observe from the pair is only either a black hole with positive energy or a wormhole mouth opening to the observer's universe, interpreting that the black hole in the "*other universe*" has negative mass without violating the positive-energy theorems [14,15]. This scenario gives spacetime a different structure. Indeed it is well known that flat spacetime cannot spontaneously generate a black hole, otherwise energy conservation would be violated. In other terms we cannot compare spacetimes with different asymptotic behaviour [16]. The different boundary

conditions reflect on the fact that flat space is not periodic in euclidean time which means that the temperature is zero. On the other hand a black hole with an imaginary time necessitates periodicity, but this implies a temperature different from zero. Then, unless flat spacetime has a temperature T equal to the black hole temperature, there is no chance for a transition from flat to curved spacetime. This transition is a decay from the false vacuum to the true one [10–13]. However, taking account a pair of neutral black holes living in different universes, there is no decay and more important no temperature is necessary to change from flat to curved space. This could be related with a vacuum fluctuation of the metric which can be measured by the Casimir energy.

IV. CASIMIR ENERGY

One can in general formally define the Casimir energy as follows

$$E_{Casimir}\left[\partial\mathcal{M}\right] = E_0\left[\partial\mathcal{M}\right] - E_0\left[0\right], \tag{9}$$

where E_0 is the zero-point energy and $\partial\mathcal{M}$ is a boundary. For zero temperature, the idea underlying the Casimir effect is to compare vacuum energies in two physical distinct configurations. We can recognize that the expression which defines quasilocal energy is formally of the Casimir type. Indeed, the subtraction procedure present in eq.(3) describes an energy difference between two distinct situations with the same boundary conditions. However, while the expression contained in eq.(3) is only classical, the Casimir energy term has a quantum nature. One way to escape from this disagreeable situation is the induced gravity point of view discussed in Ref. [17] and refs. therein. However, in those papers the subtraction procedure in the energy term is generated by the zero point quantum fluctuations of matter fields. Nevertheless, we are working in the context of pure gravity, therefore quasilocal energy has to be interpreted as the zero loop or tree level approximation to the true Casimir energy. To this end it is useful to consider a generalized subtraction procedure extended to the volume term up to the quadratic order. This corresponds to the semiclassical

approximation of quasilocal energy. What are the possible effects on the foam-like scenario? Suppose we enlarge this process from one pair to a large but fixed number of such pairs, say N. What we obtain is a multiply connected spacetime with N holes inside the manifold, each of them acting as a single bifurcation surface with the sole condition of having symmetry with respect to the bifurcation surface even at finite distance. Let us suppose the interaction between the holes can be neglected, i.e., let us suppose that the total energy contribution is realized with a coherent summation process. This is equivalent to saying that the wave functional support (here, the semiclassical WDW functional) has a finite size depending only on the number of the holes inside the spacetime. It is clear that the number of such holes cannot be arbitrary, but is to be related with a minimum size of Planck's order. However this amazing mechanism is still to be verified. One possibility is the computation of

$$\Gamma = \frac{P_{N-holes}}{P_{flat}} \simeq \frac{P_{foam}}{P_{flat}},$$

via the semiclassical WDW functional. Another way is to be found in the generalized subtraction procedure, that is, in the evaluation of the Casimir energy. Nevertheless we obtain, in both cases, a semiclassical result from which we can extract hints on the possibility of obtaining a foam-like scenario.

V. ACKNOWLEDGMENTS

I wish to thank Dott J. Pearson for her invaluable linguistic advice.

REFERENCES

[1] J.A. Wheeler, Ann. Phys. **2** (1957) 604.

[2] S. Coleman, Nucl. Phys. B **310** (1988) 643.

[3] F.J. Ernst, J. Math. Phys., **17** (1976) 515.

[4] M.A. Melvin, Phys. Lett. **8** (1964) 65.

[5] S. Nariai, *On some static solutions to Einstein's gravitational field equations in a spherically symmetric case.* Science Reports of the Tohoku University, **34** (1950) 160; S. Nariai, *On a new cosmological solution of Einstein's field equations of gravitation.* Science Reports of the Tohoku University, **35** (1951) 62.

[6] V.P. Frolov and E.A. Martinez, *Action and Hamiltonian for Eternal Black Holes,* Class.Quant.Grav.13 :481-496,1996, gr-qc/9411001.

[7] S. W. Hawking and G. T. Horowitz, *The Gravitational Hamiltonian, Action, Entropy and Surface Terms*, Class. Quant. Grav. **13** 1487, (1996), gr-qc/9501014.

[8] R. Bousso and S.W. Hawking, *The probability for primordial black holes.* Phys. Rev. D **52**, 5659 (1995), gr-qc/9506047; R. Bousso and S.W. Hawking, *Pair creation of black holes during inflation.* Preprint no. DAMTP/R-96/33, gr-qc/9606052.

[9] E.A. Martinez, *Entropy of eternal black holes.* To appear in the proceedings of the Sixth Canadian Conference on General Relativity and Relativistic Astrophysics, gr-qc/9508057.

[10] S. Coleman, Nucl. Phys. B **298** (1988), 178.

[11] P. Ginsparg and M.J. Perry, Nucl. Phys. B **222** (1983) 245.

[12] D.J. Gross, M.J. Perry and L.G. Yaffe, Phys. Rev. D **25**, (1982) 330.

[13] P.O. Mazur Mod. Phys. Lett. A **4**, (1989) 1497.

[14] R. Garattini, *Energy computation in wormhole background with the Wheeler-DeWitt operators.* In *Second Meeting on Constrained Dynamics and Quantum Gravity*, Nucl. Phys. B (Proc. Suppl.) 57 (1997) 316-319, gr-qc/9701060; R. Garattini, *Wormholes and Black Hole Pair Creation*, gr-qc/9609009.

[15] P.F. Gonzalez-Diaz, *The Schwarzschild Black Hole Pair.* In *Topics in Quantum Field Theory. Modern Methods in Fundamental Physics,* ed. D.H. Tchrakian (World Scientific, Singapore, 1995) p. 142, gr-qc/9503011; P.F. Gonzalez-Diaz, *Quantum black-hole kinks*, Grav. Cosmol. 2 (1996) 122-142, gr-qc/9606036.

[16] E. Witten, Nucl. Phys. B **195 (**1982) 481.

[17] F. Belgiorno and S. Liberati, *Black Hole Thermodynamics, Casimir Effect and Induced Gravity*, Gen. Rel.Grav (29) 1997 1181-1194, gr-qc/9612024.